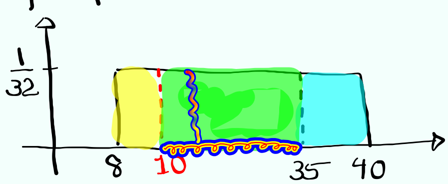
Statistics

Lecture 10

(SG17)

Consider a uniform Prob. dist. for all values from 8 to 40.

1) Graph & label.



2) $P(x=10)=0$

3) $P(x<10 \text{ OR } x>35)$

$=1-P(10<x<35)$

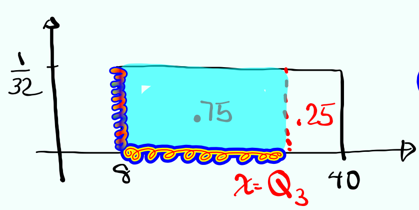
$=1-(35-10)\cdot\frac{1}{32}$

$=1-\frac{25}{32}=\boxed{\frac{7}{32}}$

4) Find $x=Q_3$

75% below

25% above

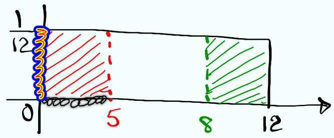


$(x-8)\cdot\frac{1}{32}=.75$

$x-8=32(.75)$

$x=8+32(.75)=\boxed{32}$

The wait time at a local coffee shop has a uniform Prob. dist with max. wait time of 12 minutes.



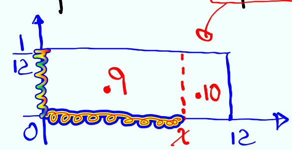
1) What is the prob. that your wait time is below 5 minutes?

$P(x<5) = (5-0) \cdot \frac{1}{12} = \frac{5}{12}$

2) What is the prob. that your wait time exceeds 8 minutes?

$P(x>8) = (12-8) \cdot \frac{1}{12} = \frac{4}{12} = \boxed{\frac{1}{3}}$

3) Find the time, rounded to whole minute, that separates the $\boxed{\text{top } 10\%}$ from the rest.



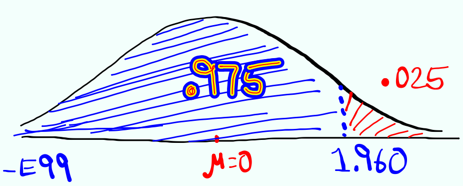
$(x-0) \cdot \frac{1}{12} = .9$

$x = 12(.9)$

$x = 10.8 \approx \boxed{11 \text{ minutes}}$

find $P(Z < 1.960)$

N(0,1)

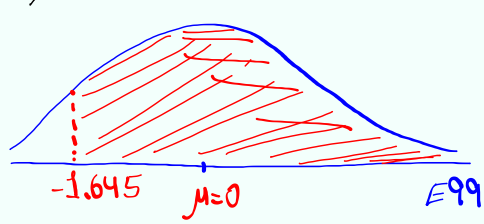


$= \text{normalcdf}(\underset{L}{-E99}, \underset{U}{1.960}, \underset{\mu}{0}, \underset{\sigma}{1}) = \boxed{.975}$

(-)

$\boxed{2nd}\boxed{,}$

find $P(Z > -1.645)$

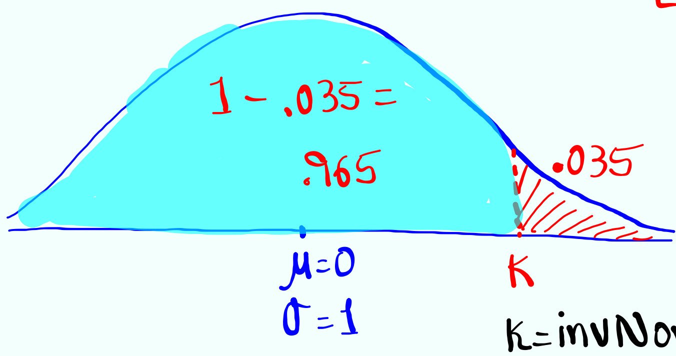


(-)

$= \text{normalcdf}(-1.645, E99, 0, 1) = \boxed{.950}$

σ=1

find k such that $P(Z > k) = .035$ Right Area



Left Area

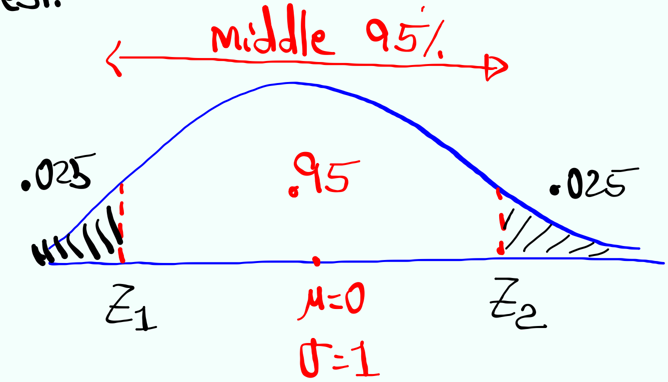
$k = \text{invNorm}(.965, 0, 1)$

$\approx \boxed{1.812}$

find two Z-values that separate the middle 95% from the rest.

$1 - .95 = .05$

$\frac{.05}{2} = .025$



$Z_1 = \text{invNorm}(.025, 0, 1) = \boxed{-1.960} \approx -2$

$Z_2 = \text{invNorm}(.975, 0, 1) = \boxed{1.960} \approx 2$

Normal Prob. dist.

1) Use x , $P(x=c)=0$

2) Bell-Shape, Symmetric, total Area $=1$

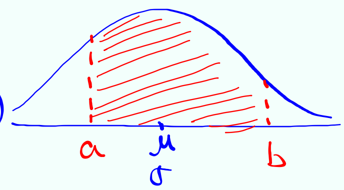
3) Mean = Mode = Median

4) Mean μ & Standard deviation σ are both given in the Problem.

5) $\boxed{P(a<x<b)}$ is the Corresponding area within the normal curve.

How to find it:

normalcdf(L, U, μ, σ)



$N(\mu, \sigma)$

Normal — Mean — Standard Deviation

For reverse invNorm(Left Area, μ, σ)

Given $N(82, 6)$

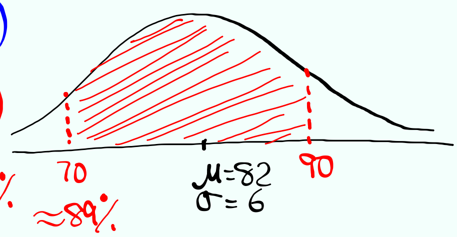
Normal Prob. Dist. — μ — σ

Find $P(70<x<90)$

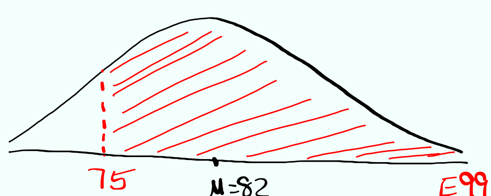
$= \text{normalcdf}(70, 90, 82, 6)$

$= \boxed{.886} \approx 88.6\%$

$\approx 89\%$



Find $P(x>75)$

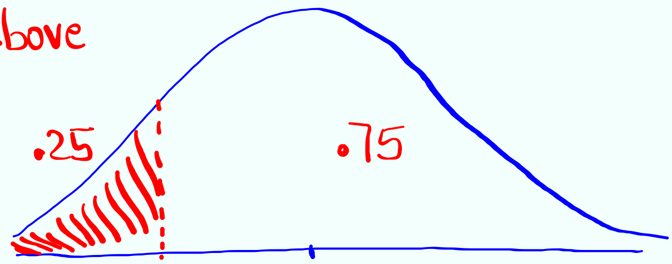


$= \text{normalcdf}(75, E99, 82, 6)$ $\sigma=6$ $= \boxed{.878}$

Find $x = Q_1$, Round to a whole #.

25% Below

75% above



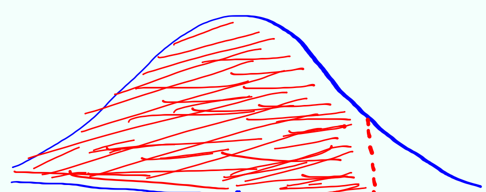
$x = Q_1 = \text{invNorm}(.25, 82, 6)$

$\approx \boxed{77.953} \approx \boxed{78}$

Ages of all students at college are normally distributed with the mean of 32 Yrs and standard deviation of 7. $N(32, 7)$

If we randomly select one student, find the prob. that his/her age (x) is below 40 Yrs.

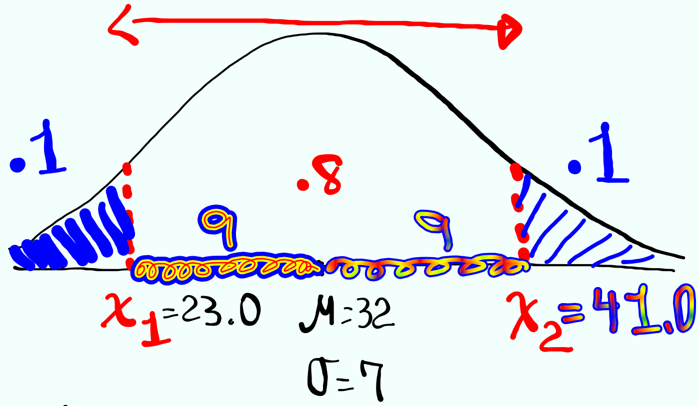
$P(x < 40)$



$= \text{normalcdf}(-E99, 40, 32, 7) \approx \boxed{.873}$

Find two ages, round to 1-decimal place, that separate the middle 80% from the rest.

$$\frac{1-.8}{2} = .1$$



$x_1 = \text{invNorm}(.1, 32, 7) \approx \boxed{23.0}$

$x_2 = \text{invNorm}(.9, 32, 7) \approx \boxed{41.0}$

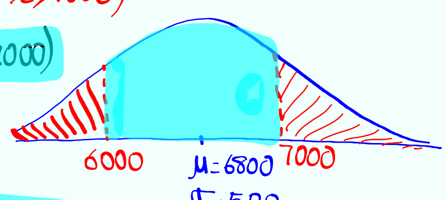
Salaries of all nurses in LA County has a normal dist with mean monthly Salary of \$6800 and standard deviation of \$500.

N(6800, 500)

If we randomly select one nurse, find the Prob. that his/her monthly Salary (x) is below \$6000 or above \$7000

$P(x<6000 \text{ OR } x>7000)$

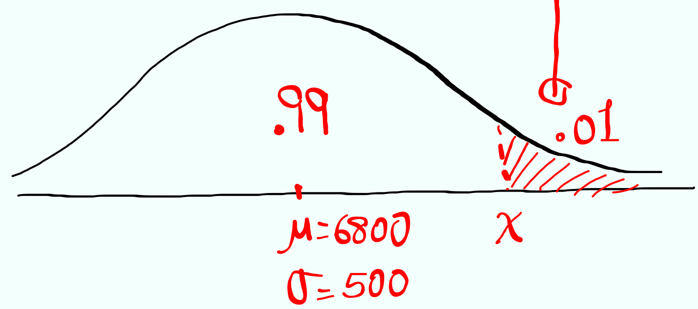
$= 1 - P(6000 < x < 7000)$



$= 1 - \text{normalcdf}(6000, 7000, 6800, 500)$

$= \boxed{.399} \approx 39.9\% \approx 40\%$

Find a Salary, round to whole #, that separates the $\boxed{\text{top } 1\%}$ from the rest.



$x = \text{invNorm}(.99, 6800, 500) \approx 7963.174$

$\approx \boxed{\$7963}$

SG 17 & 18 ✓

Clear all lists $\boxed{\text{2nd}}$ $\boxed{+}$ $\boxed{\text{4:clearAllLists}}$ $\boxed{\text{Enter}}$ SG 19

Reset all lists $\boxed{\text{STAT}}$ Edit $\boxed{\text{5:SetUpEditor}}$ $\boxed{\text{Enter}}$

Store 2, 4, 6, 8 in L1.

Use $\boxed{\text{1-Var Stats}}$ with L1 only to find

$\boxed{\mu = 5}$ $\sigma = 2.236$ $\boxed{\sigma^2 = 5}$

Take all Samples of $\boxed{\text{Size } 2}$ with $\boxed{\text{replacement}}$

2,2	2,4	2,6	2,8
4,2	4,4	4,6	4,8
6,2	6,4	6,6	6,8
8,2	8,4	8,6	8,8

Find $\bar{x}$ of each Sample

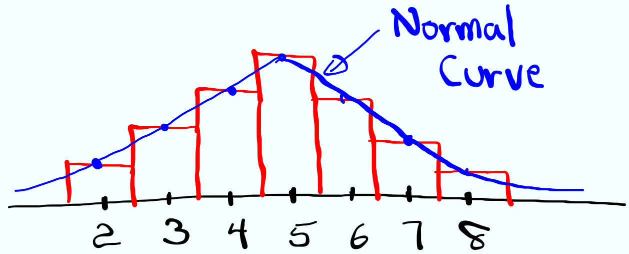
2	3	4	5
3	4	5	6
4	5	6	7
5	6	7	8

16 means

$\bar{x}$	$P(\bar{x})$
2	1/16
3	2/16
4	3/16
5	4/16
6	3/16
7	2/16
8	1/16

$\bar{x}$	$P(\bar{x})$
2	1/16
3	2/16
4	3/16
5	4/16
6	3/16
7	2/16
8	1/16

Draw Prob. Dist. Histogram



$\bar{x} \to L2$, $P(\bar{x}) \to L3$

Use $\boxed{\text{1-Var Stats}}$ with L2 & L3

Find

$\boxed{\mu = 5}$

$\sigma = 1.581$

$\boxed{\sigma^2 = \frac{5}{2}}$

Clear all lists

Store 2, 4, 6, 8, 10 in L1.

Use $\boxed{\text{1-Var Stats}}$ with L1 to Find

$\boxed{\mu = 6}$ $\sigma = 2.828$ $\boxed{\sigma^2 = 8}$

Take all Samples of $\boxed{\text{Size 2}}$ with replacement.

2,2	2,4	2,6	2,8	2,10
4,2	4,4	4,6	4,8	4,10
6,2	6,4	6,6	6,8	6,10
8,2	8,4	8,6	8,8	8,10
10,2	10,4	10,6	10,8	10,10

Find $\bar{x}$ of each Sample

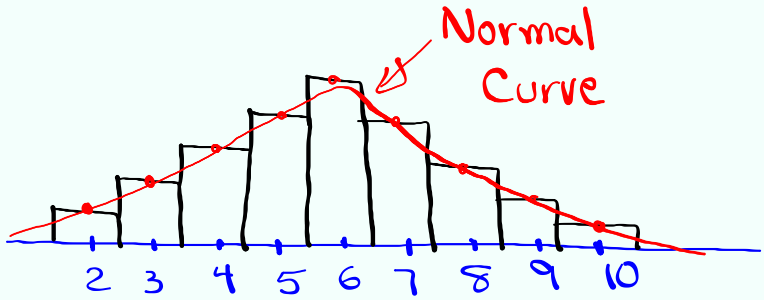
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8
5	6	7	8	9
6	7	8	9	10

25 means

$\bar{x}$	$P(\bar{x})$
2	1/25
3	2/25
4	3/25
5	4/25
6	5/25
7	4/25
8	3/25
9	2/25
10	1/25

$\bar{x}$	$P(\bar{x})$
2	1/25
3	2/25
4	3/25
5	4/25
6	5/25
7	4/25
8	3/25
9	2/25
10	1/25

Draw Prob. dist. histogram



$\bar{x} \to L2$, $P(\bar{x}) \to L3$

Use $\boxed{\text{1-Var Stats}}$ with L2 & L3

$\boxed{\mu = 6}$ $\sigma = 2$ $\boxed{\sigma^2 = 4 = \frac{8}{2}}$

Central Limit theorem

CLT

$\boxed{\mu_{\bar{x}} = \mu}$

$\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$

$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$

Suppose exam scores of all exams has $\mu = 82$ and $\sigma = 8$. If we take all samples of size 4,

$\mu_{\bar{x}} = \mu = 82$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{8}{\sqrt{4}} = \frac{8}{2} = 4$

Suppose Salaries of all nurses in LA county has $\mu = 6800$ & $\sigma = 500$.

If we randomly select all Samples of 16 nurses,

$$\mu_{\bar{x}} = \mu = 6800 \qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{500}{\sqrt{16}} = \frac{500}{4} = 125$$

Ages of all teachers are N.D. with $\mu = 52$ Yrs and $\sigma = 6$ Years. $N(52, 6)$

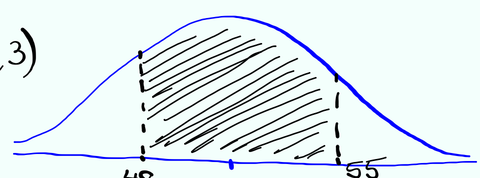
If we randomly Select all Samples of Size 4, ($n = 4$)

find the prob. that their mean age ($\bar{x}$) is between 48 and 55 Yrs.

$P(48 < \bar{x} < 55)$

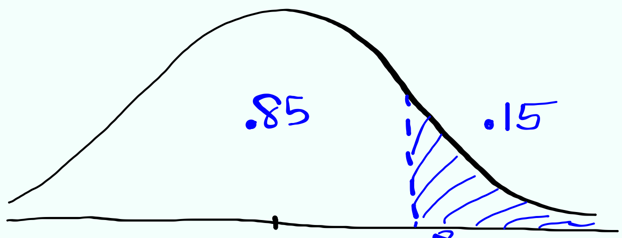
$= \text{normalcdf}(48, 55, 52, 3)$

$= \boxed{.750}$



CLT $\begin{cases} \mu_{\bar{x}} = \mu = 52 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{6}{\sqrt{4}} = 3 \end{cases}$

Find $\bar{x}$ for randomly selected all samples of size 5 that separate the $\boxed{\text{top } 15\%}$ from the rest. Round to whole #.



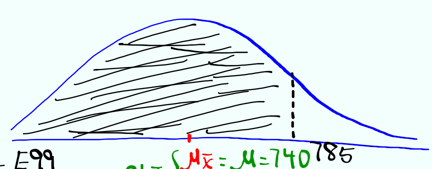
CLT $\begin{cases} \mu_{\bar{x}} = \mu = 52 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{6}{\sqrt{5}} \end{cases}$

$\bar{x} = \text{invNorm}(.85, 52, 6/\sqrt{5}) \approx 54.781 \approx \boxed{55}$

Credit Scores are normally dist. with $\mu = 740$ & $\sigma = 50$. $N(740, 50)$

If we randomly choose samples of $\boxed{\text{Size } 8}$ ($n = 8$) applicants, find the prob. that $\boxed{\text{their mean}}$ ($\bar{x}$) Credit Scores is $\boxed{\text{below } 785.}$

$P(\bar{x} < 785)$



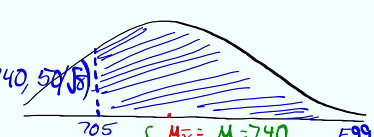
CLT $\begin{cases} \mu_{\bar{x}} = \mu = 740 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{50}{\sqrt{8}} \end{cases}$

$= \text{normalcdf}(-E99, 785, 740, 50/\sqrt{8}) \approx \boxed{.995}$

$P(\bar{x} > 705)$

$= \text{normalcdf}(705, E99, 740, 50/\sqrt{8})$

$= \boxed{.976}$



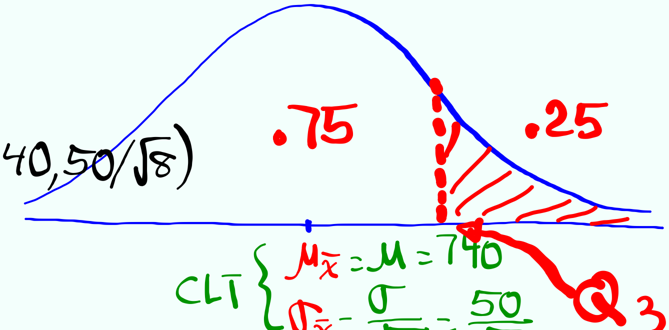
CLT $\begin{cases} \mu_{\bar{x}} = \mu = 740 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{50}{\sqrt{8}} \end{cases}$

find $\bar{x} = Q_3$, round to whole # with Same Sample Size.

$\bar{x} = Q_3$

$= \text{invNorm}(.75, 740, 50/\sqrt{8})$

$\approx \boxed{752}$



CLT $\begin{cases} \mu_{\bar{x}} = \mu = 740 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{50}{\sqrt{8}} \end{cases}$

SG 19 & 20 ✓